U.S. COAST AND GEODETIC SURVEY—...GRID SYSTEM FOR PROGRESSIVE MAPS IN THE U.S.

PREFACE.

In July, 1918, Maj. R. C. Kuldell, of the Corps of Engineers, consulted with certain officials of the United States Coast and Geodetic Survey in regard to the possibility of devising a grid system for military surveys and maps along the coast of the United States, to be used in connection with artillery defense maps. The system adopted was the one proposed by Mr. Bowie and is described in the first part of this publication.

Shortly after the adoption of this system, Mr. Bowie was commissioned a major of Engineers and served from August 17, 1918, to February 28, 1919, in the Division of Military Mapping of the Corps of Engineers, Washington, D. C. The descriptive part of this report was prepared by him while he was in the Army.

The derivation of formulas with which the tables were computed was made by Mr. Adams, who had also direct charge of all the computations connected with the preparation of the tables which appear herein. He was assisted in the computations by a number of computers of the Coast and Geodetic Survey and by enlisted men of the Four hundred and seventy-second Engineers, who were assigned to the office of the Coast and Geodetic Survey to aid in the computation and interpolation of the tables. Especial credit is due Sergt. T. F. Shea, who was one of the detail from the Four hundred and seventy-second Engineers and who, after his discharge from the Army, became a geodetic computer in the Coast and Geodetic Survey.

The table given in the publication consists of plane coordinates of five-minute intersections of latitude and longitude.

If anyone wishes to go more fully into the discussion of the mathematical theory of the polyconic projection that forms the basis of this table, a complete development of the same will be found in Special Publication No. 57, The General Theory of Polyconic Projections, by Mr. Adams. Copies of this publication can be procured by application to the Superintendent of the Coast and Geodetic Survey, or to the Superintendent of Documents, Washington, D. C.

NOTE TO REVISED EDITION.

While preparing tables for the War Department, the Mathematical Tables Project undertook a check of the tables in this publication, corrections of which have been incorporated in this edition.

The work was conducted by the Federal Works Agency, Work Projects Administration of the City of New York, Sponsored by the Bureau of Standards, Dr. Lyman J. Briggs, Director; under supervision of Dr. A. N. Lowan, Director of the Project, assisted by Dr. Gertrude Blanch, Mr. Murray Pfeferman and Mr. Milton Abramowitz.

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ILLUSTRATIONS.

GRID SYSTEM FOR PROGRESSIVE MAPS IN THE UNITED STATES.

By William Bowie, *Chief, Division of Geodesy*, and Oscar S. Adams, *Geodetic Computer, U. S. Coast and Geodetic Survey.*

INTRODUCTION.

In the first few pages of this publication will be found a description of the quadrillage, or grid system, together with directions for the use of the system for progressive military mapping.

The intent when the computations were started was to compute only the necessary coordinates for the area covered by the eastern coast of the United States. It was afterwards decided to make the computations complete for a whole zone. This resulted in a table which can be used for the whole area of the United States proper, as the same values can be used for all of the seven zones by merely changing the designation of the longitude to fit the particular zone needed. The tables now cover an area which is 9° wide in longitude and 21° 20′ in length in latitude. There are several small areas, notably in eastern Maine, southern Florida, and Texas, for which extensions of the tables beyond the above limits are provided.

The seventy-third meridian is the central meridian of the first zone. The zones extending across country have their central meridians 8° apart, thus providing an overlap of 1° in longitude which will make it possible to have two systems of grid lines on a map at the junction of two zones in order that the map may be used in connection with other maps of either of the zones.

It is impossible to have one grid system to cover the whole area of the United States without dividing it into zones, if it is desired to have a small allowable limit of error in the system. The plan adopted was to have such a width of zone as would make the errors introduced in the maps negligible for practically all military purposes.

PROJECTION.

The earth's surface is spherical, and in order that it may be represented on a map it is necessary to show it on a plane surface. This necessarily means that no portion of the earth's surface can be shown in absolute accuracy on a map. There are various methods of approximating the surface of the earth on the plane, one of the most common of which is known as the polyconic projection.

A projection is simply a system of lines on a map representing imaginary lines on the earth's surface. The almost universal plan for these lines upon the earth's surface is to adopt a system of spherical coordinates based upon the plane through the poles and the observatory at Greenwich, England, and the plane cutting the earth's axis at right angles midway between the poles. The first plane is called the initial meridian, to which longitude or angular distances east and west are referred. The second plane is called the Equator, to which angular distances north and south or latitude are referred.

It is possible to determine by astronomic methods the longitude and latitude of any point on the earth's surface. The problem is how to show the location of some such point on a map with relation to the initial meridian and the Equator.

There are various ways of showing meridians and parallels of latitude on a piece of paper, with relation to which various points on the earth's surface can be plotted after their latitude and longitude have been determined. As mentioned above, one of the most generally used projections is what is called the polyconic projection. This is the one in which every parallel of latitude appears on the map as the developed circumference of the base of a right cone tangent to the sphere or spheroid on that parallel. The central meridian of this projection for any map will appear as a straight line, while all other meridians will appear slightly concave toward it. The parallels for this projection on any map will appear as arcs of circles of different radii with the centers of the arcs of circles on the central meridian produced beyond the limits of the map except in the case of the region at the North Pole or the South Pole. The Equator alone of all the parallels will be represented on the map by a straight line and all of the other parallels will be convex toward it. The intersection of the various meridians and parallels for the polyconic projection will differ very little from right angles when the map covers a limited area in the east-and-west direction. The distance along the central meridian will be true to the scale of the map. This will also be so in the case of the distance along each of the arcs of the parallels, but diagonal distances and north-and-south distances will be slightly in error, the error increasing with the distance from the central meridian.

It will be readily seen that when a section of the earth's surface, which is spherical, is flattened out to the plane of the map, there must be some distortion in the distances as given by the map. If the central meridian is held true to scale, as is the case in the polyconic projection, and also the parallels are true to scale, then there must be some distortion as the edges of the map are approached. This

will result in a sort of stretching or elongating of the area in a north-and-south direction.

For small areas the error in a polyconic projection is infinitesimal and can be ignored for even the very highest grade of map, but when the area approaches the dimensions of a number of square degrees, then the distortion at the east and west edges of the map may be material in size. In an area as large as the United States the errors in a north-and-south direction along the Atlantic coast, and also along the Pacific coast, are as great as 6 per cent. This, of course, would make the polyconic projection a very poor one for an area of such a large extent.

With an area no greater than 10° of longitude in width, the error of a polyconic projection will be negligible so far as the scale of the map is concerned. For instance, at a distance of 5° east or west from the central meridian any distance in a north-and-south direction, as scaled from the chart, will differ from the true distance on the earth's surface by only 0.22 of 1 per cent. When it is realized that the hygrometric condition will expand or contract a map by as much as 1 per cent, it is seen that the error of the polyconic projection is negligible for this width.

PROGRESSIVE SPECIAL MILITARY MAP.

The polyconic projection shows the meridians and parallels as curved lines, except for a central meridian. With relation to these curved lines, points can be plotted whose geographic positions in latitude and longitude are known, but it is a difficult matter, requiring considerable time, to compute the distance and direction between two such points shown on a polyconic-projection map. It is, therefore, not desirable to have such a projection on a map that is to be used for military purposes, where distances and directions between objects must be computed in a very short time. Such a case is in the use of artillery. The orientation officer of a battery may wish to know quickly the distance and bearing from his gun to some point occupied by the enemy in order that the enemy may be fired upon. The only quick method of obtaining this distance is by a system of rectangular coordinates. The distance between the two objects whose coordinates are known would be the square root of the sum of the squares of the difference of the x-coordinates and of the y-coordinates of the two objects. These differences make it possible to compute quickly the angle between one of the grid lines and the line joining the objects.

A system of rectangular coordinates could be used to cover any local area without regard to any other locality, but this makes an awkward situation when the maps of any two areas adjoin. The boundary between the two areas would be on different systems, and much confusion would result. In order to obviate this there has been adopted a plan by which a single system of grid coordinates covers the Atlantic coast from some point on the coast of North Carolina to the northeastern part of the coast of Maine.

GRID TABLES.

In order that such a system of coordinates may be properly coordinated with positions as given in longitude and latitude, it is found necessary to have a polyconic projection covering a zone 9° wide in longitude and of indefinite extent in latitude. The central meridian of this projection is coincident with the seventy-third meridian. A rectangular or grid system of squares 1000 yards on a side is then constructed over the whole area covered by the projection. The intersection of the seventy-third meridian of longitude and the parallel of 40° 30′ of latitude, is the initial point of the grid system. All computations were made from that point, and a north-and-south line of the grid coincides with the seventy-third meridian and an east-and-west line of the grid is tangent to the parallel of latitude of 40° 30′ at its intersection with the seventy-third meridian.

ORIGIN OF GRID COORDINATES.

The computation consisted of determining the grid coordinates of all intersections of minutes of longitude with minutes of latitude within the area covered by the polyconic projection referred to above. After the computations were made, a constant was added to each of the x coordinates and another constant to each of the y coordinates in order to make all of the coordinates positive within the area of the projection. The point selected as the arbitrary origin is 1 000 000 yards west and 2 000 000 yards south of the intersection of the seventy-third meridian of longitude with the parallel 40° 30′ of latitude.

LIMITING MERIDIANS OF THE ZONES.

It is impossible to have a progressive map over a large area extending both east and west and north and south without serious distortions in the projection. As explained above it was found advisable to use the grid system within a zone restricted to 9° of longitude, but extending indefinitely in latitude. The tables used for the special military maps were computed for the zone extending 4° 30′ of longitude in both the east and the west directions from longitude 73°, and from latitude 28° to latitude 49° 10′. Over this whole zone the grid is constructed with straight lines. Extensions of the table are made to cover the area in Maine to the eastward of meridian

68° 30′, the small area above latitude 49° at the Lake of the Woods near longitude 95°, and for Florida and Texas below latitude 28°.

The same tables can be used for any other zone having the same latitudes by simply changing the degrees of longitude. When this is done it is possible to have the whole area of the United States covered. For instance, the meridians in the tables which are now 68° 30′ to 77° 30′ could be changed to 77° 30′ to 86° 30′, when they could be used for placing the grid over the zone falling between those meridians.

It is not practicable, however, to change the tables exactly 9°, for if this were done the maps at the junction of the two zones would not have continuous grid lines; that is, a map at the edges of one zone could not be exactly connected with a map having the same latitudes at the edges of the other zone and have the grid system continuous from one to the other. The grid lines of the two maps would make decided angles with each other.

In order to avoid this condition it is necessary to have an overlap of the two zones. This is done by changing all of the longitudes of the tables as given for the most eastern zone by 8°. This provides an overlap of 1° of longitude, and the topography within this overlapping area can be shown on two sets of maps—one on each grid system—thus making it possible to have progressive maps for each of the zones, or the two grid systems can be placed in different colors on the same map. The amount of overlapping of maps will be decided at the Office of the Chief of Engineers.

By changing the longitudes of the tables by 16° a third zone is obtained, and so on across the country in multiples of 8° of longitude.

The following table shows the designation of the several zones across the United States, with their central meridians and the meridians which limit the zones:

Fire-control zones.

Designation.	Central meridian.	Limiting meridians.
A	73°	68° 30′– 77° 30′
B	81°	76° 30′– 85° 30′
C	89°	84° 30′– 93° 30′
D	97°	92° 30′–101° 30′
E	105°	100° 30′–109° 30′
F	113°	108° 30′–117° 30′
G	121°	116° 30′–125° 30′

The only exception, so far as limiting meridians are concerned, is the strip over Maine to the eastward of longitude 68° 30′ which is included in the grid tables for zone A, though it is more than 4° 30′ to the eastward of the central meridian of the zone.

The zones are shown graphically in figure 1.

The one-hundred-thousand-yard grid lines are shown for zone C in figure 2.

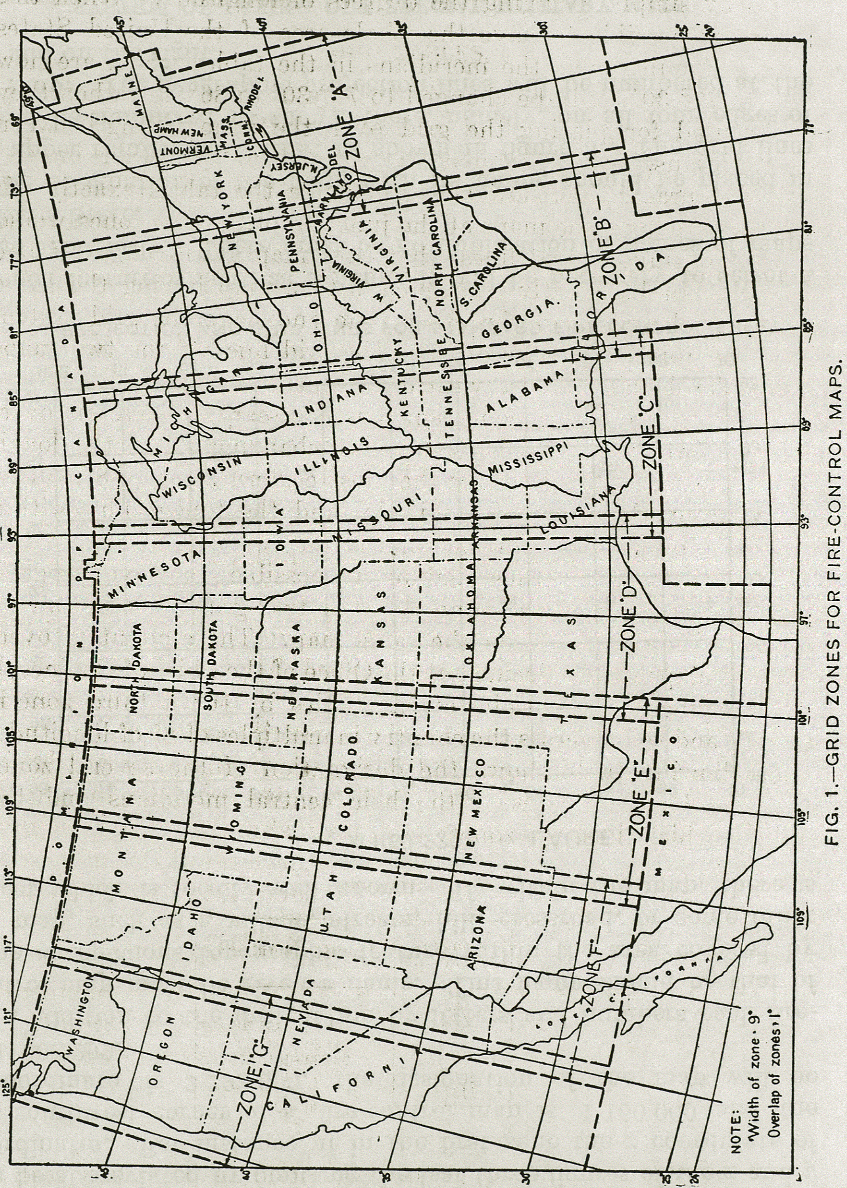

FIG. 1.—GRID ZONES FOR FIRE-CONTROL MAPS.

DESIGNATION OF SPECIAL MILITARY MAPS.

It is necessary that each special military map be given the letter of the zone within which it falls. Immediately following the letter should be given the coordinates of the southwest corner of the map.

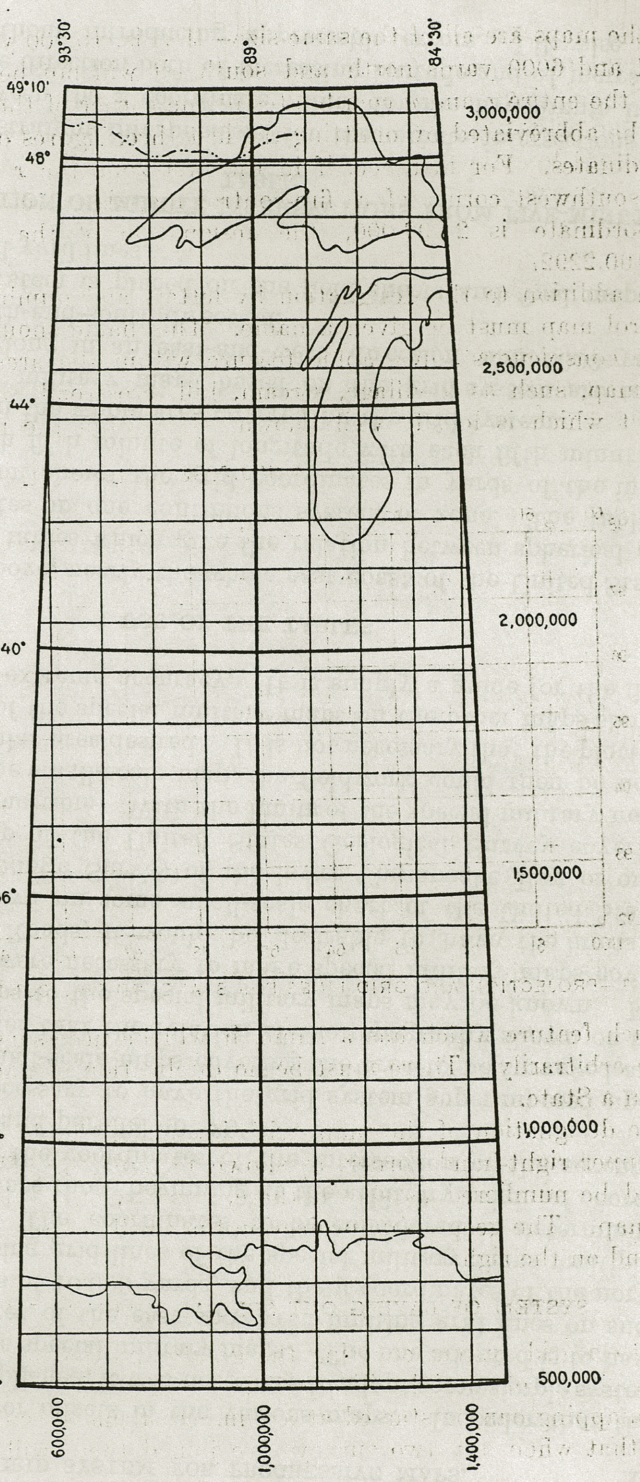

FIG. 2.—SPECIAL MILITARY MAP OF ZONE C

As the maps are all of the same size—that is, 10 000 yards east and west and 6000 yards north and south—it will not be necessary to give the entire x and y coordinates of the corner of the map. They can be abbreviated by omitting the last three figures of the x and y coordinates. For instance, if in the first zone the x coordinate of the southwest corner of a fire-control map is 1 160 000 and the y coordinate is 2 292 000, the designation of the map will be A 1160.2292.

In addition to the designation by letters and numbers each fire-control map must be given a name. This name should be that of some conspicuous topographic feature within the area covered by the map, such as a village, stream, hill, crossroad, or some other object which is locally well known. If, within the map, there is

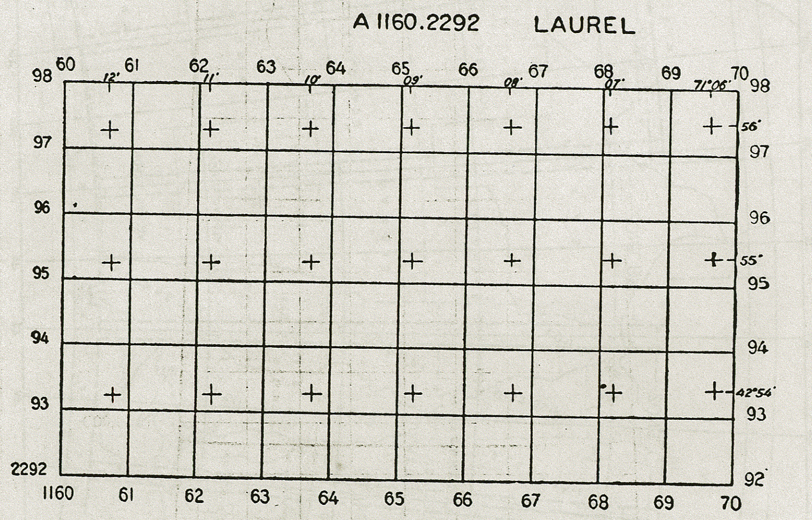

FIG. 3.—PROJECTION AND GRID LINES FOR STANDARD FIRE-CONTROL SHEET.

no such feature which has a name, it will be necessary to select a name arbitrarily. There must be no duplication of names of maps within a State.

The designation of the map, with its name, should be placed in the upper right-hand corner, as shown in figure 3. The grid lines should be numbered as shown there, namely, on all four edges of the map. The geographic projection lines will be numbered at the top and on the right.

SYSTEM OF LAYING OUT SPECIAL MILITARY MAPS.

With a grid system covering the greater portion of the Atlantic coast, a series of special military maps can be laid out in such a way that mapping can be started at a number of places with the assurance that when any two surveys are joined there will not be any

overlaps, gaps, or offsets in the various maps. In order that this might be accomplished, it was necessary to adopt some simple system of laying out the special military maps. The one adopted is to have the x coordinates of the east and west limiting grid lines on such a map multiples of 10 000 yards, and the y coordinates of the north and south limiting grid lines of the special military maps multiples of 6000 yards. The coordinates to be considered in determining these multiples are those beginning at the arbitrary origin of coordinates, and not the coordinates of the intersection of the seventy-third meridian and parallel 40° 30′.

It will be necessary to have the grid system superimposed upon one or more small-scale maps covering the area to be surveyed and mapped, in order that the relation of the topography shown on the small-scale maps to the special military maps may be known. For instance, if it were necessary to make special military maps covering Hampton Roads, it would be desirable to have the limits of those maps shown on some small-scale chart of the United States Coast and Geodetic Survey of the lower Chesapeake Bay or on a small-scale map of the United States Geological Survey covering the region in question. With the limits of the special military maps laid out on the small-scale map, the fieldwork could then be done for any particular area desired. It is not necessary that the plotting of the limits of the special military maps on the other maps should be done with extreme accuracy. It is simply a guide for the field operations.

USE OF THE TABLES.

As stated above, nearly the whole east coast of the United States is covered by tables which give the relation between spherical and grid coordinates on one continuous system or zone. The table in this publication shows the grid coordinates in yards of the intersection of each fifth minute of longitude with each fifth minute of latitude within the whole area covered by the grid system.

The special military maps based on the grid system are to be 10 000 yards long in an east-and-west direction and 6000 yards deep in a north-and-south direction.

The grid system is placed on the field sheets and final maps as even thousand-yard lines.

INTERPOLATION OF MINUTE INTERSECTIONS FROM FIVE-MINUTE TABLE.

Linear or straight interpolation for the minute intersections can be made between the x coordinate values for the five-minute intersections in the direction both of increasing latitude and of increasing longitude without introducing appreciable errors. Likewise such interpolations can be made between the y values in the direction of increasing latitude, but in the direction of increasing longitude an error as great as 1 yard may be introduced in the value of y by linear interpolation.

If the work in hand is such that this error is appreciable, the y values should be interpolated in the longitudinal direction in the following manner:

The departure of the parallel from the x grid line tangent to it at its intersection with the central meridian should be interpolated as the squares of the distances out from the central meridian. For example, if we wish to interpolate the y value for latitude 37° 05′ and longitude 75° 02′, we should proceed in the following manner:

$$\left.\begin{array}{l}37^\circ\ 05'\\73^\circ\ 00'\end{array}\right\}y=1585214.0$$

$$\left.\begin{array}{l}37^\circ\ 05'\\75^\circ\ 00'\end{array}\right\}y'=1587260.3$$

$$y''=y'-y=2046.3$$

75° 00′ is 2° or 120′ out from central meridian

75° 02′ is 2° 02′ or 122′ out from central meridian.

$$y \text{ for } 75^\circ\ 02'=1585214.0+\left(\frac{122}{120}\right)^2\times 2046.3$$

$$=1587329.1.$$

Corresponding values are

1587329.0 from minute table

or 1587329.9 by linear interpolation.

After the x and y coordinates have been interpolated along two contiguous five-minute parallels, as explained above, then straight interpolation along the meridian between these values can be made for intermediate parallels.

THE KILOMETRIC GRID.

The grid tables computed in yards can be used to place a kilometric grid on a map. This can be done by converting to meters the yard coordinates of one of the thousand-yard intersections on the map. With these meter coordinates points can be plotted which will have values to even kilometers. Through these points kilometer grid lines will be drawn parallel to the regular yard grid lines. For instance, if the x and y coordinates converted to meters had values at the southwest corner of the map of 953 624 and 1 242 719, respectively, then the first north-and-south kilometer grid line would be 376 meters east of the corner and the first east-and-west kilometer grid line would be 281 meters north of the corner. After these

two points of the kilometric system have been found, north-and-south and east-and-west lines can be drawn through them parallel to the thousand-yard grid lines. After two kilometric grid lines have been placed upon the map, the whole map can easily be covered by the kilometric grid system. This is done by simply plotting along the east-and-west line distances of 1, 2, 3, etc., kilometers from the north-and-south kilometer grid line. Similarly the east-and-west kilometer grid lines can be laid off on the north-and-south line.

If the map has been at all distorted since the yard grid was placed upon it, it would be well to convert the coordinates of several of the thousand-yard grid intersections into meters, and the intermediate grid lines of the kilometric system can then be interpolated. In this way the effect of any distortion of the map is minimized.

METHOD OF PLACING THE SPHERICAL PROJECTION ON SPECIAL MILITARY MAPS.

Let it be supposed that a series of special military maps have been laid out on a small-scale map covering the area to be surveyed and that it is desired to place the spherical coordinates on a special military map on which the thousand-yard grid lines have been drawn.

From the small-scale map some intersection of a minute of latitude and a minute of longitude is selected which falls within the special military map in question.

The tables are entered for that particular longitude and latitude and the x and y coordinates are found. The difference between the x coordinates of the intersection and the thousand-yard north-and-south grid line just to the westward of the intersection is laid off to the eastward of that thousand-yard grid line. Likewise, the difference between the y coordinate of the intersection of the minute of latitude and longitude and of the y coordinate of the even thousand-yard east-and-west grid line just to the south of it will be the distance to be plotted to the north of that grid line. These two differences in the x and y coordinates will locate accurately on the grid system of the special military map the particular intersection of the minute in longitude and latitude. In a similar manner all the other intersections of minutes of latitude and longitude coming within the area of any special military map can be plotted.

In figure 4 let it be supposed that the grid system is already on the map as shown and that it is desired to put spherical coordinates on the same sheet in the proper relation to the grid. An inspection of the tables shows that the intersection of latitude 42° 55′ with longitude 70° falls in the lower right-hand corner of the sheet. The coordinates of this intersection are $x=1\ 267\ 834$ yards and $y=2\ 298\ 311$ yards. The position of this intersection can be obtained by plotting 834 yards east of the grid line 1 267 000 and 311 yards north of the grid line 2 298 000. The other intersections within the area of the map can be plotted in the same way.

METHOD OF PLACING GRID SYSTEM ON A MAP HAVING MERIDIANS AND PARALLELS.

If it is desired to put the grid system on a map which already has the polyconic system of projection, the operation will be as follows:

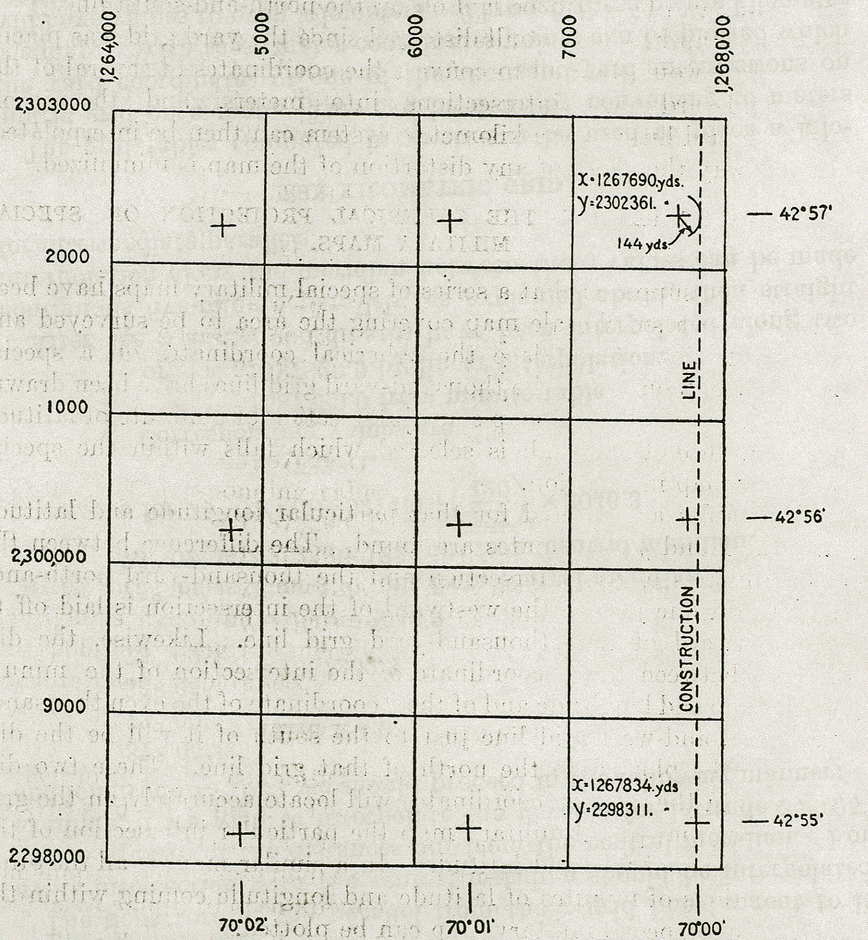

FIG. 4.—CONSTRUCTION OF GRID SYSTEM ON A MAP HAVING MERIDIANS AND PARALLELS.

Take some one intersection of a meridian and parallel in one corner of the map and enter the grid tables with these values and obtain the *x* coordinate for that intersection. Next take an intersection of the same meridian with a much higher or lower latitude. For instance, if the first intersection was in the southeast corner of the map, then take an intersection in the northeast corner, if the map is square with the meridians and parallels. Obtain from the table the *x* coordinate for this northeast intersection. Next take the difference between the *x* coordinates of these two intersections, and with this as a radius describe an arc of a circle around one of the intersections (say the northeast one). This should be to the westward of the point, if the longitude is greater than 73°, and to the eastward if less than 73°. After this arc has been drawn, place a straightedge on the other intersection of the meridian and parallel and tangent to the arc and draw a line across the map. This line will be parallel to the *y* grid lines and upon it can be constructed the grid system of the map.

Let it be supposed that in figure 4 the projection lines have been plotted and that it is later desired to superimpose the grid system. In this case the table should be entered and the *x* and *y* coordinates obtained for the intersections of latitude 42° 55′ with longitude 70° 00′ and for latitude 42° 57′ with longitude 70° 00′. The tables show a difference in the *x* coordinates for these two intersections of 144 yards. With this distance as a radius, an arc of a circle is described to the eastward of the more northern one of the two intersections. (This arc could have been described to the westward of the southern one of the two intersections.) Next draw a line through the lower minute intersection tangent to the arc of the circle which was drawn to the eastward of the upper minute intersection. This line is parallel to a north-and-south line of the grid and may be considered a construction line. The next operation is to lay off a distance of 834 yards to the westward, and at right angles to the construction line at the lower part of the sheet, and the same distance to the westward of the construction line at the upper part of the sheet. A line joining the two points thus laid off will be a north-and-south thousand-yard line of the grid system whose *x* coordinate is 1 267 000 yards.

The next operation would be to lay off the distance of 311 yards along the construction line to the southward of the intersection of latitude 42° 55′ with longitude 70° 00′. The point thus plotted will be an even thousand-yard grid line running east and west. This point can be transferred to the north-and-south grid line previously constructed by drawing a line through it at right angles to the construction line or to the north-and-south grid line.

If the map is true to scale and not distorted, the grid system can now be extended over the whole sheet; but it is probable that the scale of the map will be somewhat changed from weather conditions and there may be some distortion in the sheet. This being the case, it will be well to lay off a point on the grid system near each of the four corners of the map. The intermediate 1000-yard grid lines running both north and south and east and west can then be interpolated,

312691°—41——2

74697

and thus the effect of any error in the scale of the map or of distortion will be minimized or eliminated.

PLANE-TABLE SHEETS.

The plane table will be used for the location of the topographic features of the earth's surface. On the plane table is placed a sheet of paper on which the control points have been plotted. In order that these control points may be shown in the proper relation to each other, it is necessary to have some lines on the sheet which will represent some system of projection. In all of the special military surveys the polyconic projection will be used on the plane-table sheets, and each sheet will have a separate projection laid out upon it. There will be no relation between the projection shown on the single plane-table sheet and the general polyconic projection for the whole area of a zone.

When a plane-table sheet has a local polyconic projection placed upon it and has also the general grid system of coordinates on it, theoretically the grid lines will not be straight. The deviation of a grid line from a straight line, under this condition, will be so slight at any place within the whole zone covered by the grid system that no error will result. In fact, the deviation of the grid line from a straight line will always be within the amount of distortion of the paper due to weather conditions.

The method of laying out the polyconic projection is described in detail, and the necessary tables are given, in Special Publication No. 5 of the United States Coast and Geodetic Survey, copies of which can be obtained through the Office of the Chief of Engineers, Washington, D. C. Briefly, the process of making a polyconic projection on a plane-table sheet is to lay out the central meridian as a straight line and then enter the tables in Special Publication No. 5 and plot the intersections of minutes of latitude with minutes of longitude by means of the x and y coordinates given therein.

CONTROL POINTS ON PLANE-TABLE SHEETS.

When the plane-table sheet has been prepared by having the projection placed upon it, the next operation will be to have the control points of the triangulation and tape traverse plotted upon the sheet in their true geographic positions. These points will be used for the control of the topographic work in the field.

DETERMINATION OF DISTANCE BETWEEN POINTS FROM THEIR GRID COORDINATES.

The distance between two points whose grid coordinates are given is equal to the square root of the sum of the squares of the differences of the x coordinates and of the y coordinates. This distance will be

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true over the whole map for lines running east and west. It will be within 1 part in 1000 for lines running in any direction if within 3° of the central meridian. Beyond that limit the error in a north-and-south line will increase until it is about 1½ parts in 1000 at a distance of 4° 30′ from the central meridian.

If an accurate distance is desired, it may be obtained from the differences in the x and y coordinates, if to the y difference is applied the scale correction.

These corrections are shown in the accompanying table (see p. 31.)

The uncorrected grid distance is always greater than the true or ground distance.

TRANSFORMATION OF GEOGRAPHIC AZIMUTHS TO GRID AZIMUTHS.

Owing to the converging of meridians, any line on the earth's surface will make a different angle with the grid meridian from the angle it makes with the geographic meridian.

In order to determine easily the difference between the two azimuths, tables have been prepared from which the correction may be computed. These tables are given on page 33.

The following is an example of the computation of the difference between the geographic and grid azimuths.

Let it be desired to determine the difference between the geographic and grid azimuths at a point whose latitude is 42° 50′ and longitude 70° 18′. In the azimuth tables it is found that the azimuth correction for latitude 42° and longitude 70° is 2° 00′ 24″.8, and the correction for the same latitude and longitude 71° is 1° 20′ 17″.2. The difference between these two corrections is 40′ 07″.6. The point for which the correction is desired is in longitude 70° 18′, which may be expressed as 70°.30. The difference mentioned above multiplied by 0.30 is 12′ 02″.3. This, subtracted from 2° 00′ 24″.8, which is the correction for latitude 42° and longitude 70°, gives 1° 48′ 22″.5. It is similarly found that the correction for latitude 43° and longitude 70° 18′ is 1° 50′ 27″.6.

The difference between these two corrections for longitude 70° 18′ is 2′ 05″.1. Latitude 42° 50′ may be expressed decimally as 42°.83. The difference between the two corrections multiplied by 0.83 is 1′ 43″.8. If this is added to 1° 48′ 22″.5, the correction at 70° 18′ of longitude and 42° of latitude, the result will be 1° 50′ 06″.3, which is the difference between the grid and the geographic azimuths at a point whose latitude is 42° 50′ and longitude 70° 18′.

To the west of the central meridian of the polyconic projection on which is superimposed the grid system the grid azimuths, counted clockwise, will be larger than the geographic or spherical azimuths. Therefore, when azimuths are reckoned clockwise, the difference between the two kinds of azimuths as computed from the table must be added to the geographic azimuth to obtain the grid azimuth. Similarly, to the eastward of longitude 73°, the difference between the geographic and grid azimuths as computed from the tables, must be subtracted from the geographic azimuth in order to obtain the grid azimuth.

The tables for the special military maps apply to the whole country. Therefore, the azimuth tables can be used to obtain the difference between a geographic and a grid azimuth at any place in the United States.

The azimuth tables are extended to cover that portion of Maine to the eastward of the 68° 30′ of longitude.

Care must be taken in computing the azimuth correction in the area where two zones overlap.

TRANSFORMATION OF GRID AZIMUTHS TO GEOGRAPHIC AZIMUTHS.

The azimuth correction tables on page 33 are also used in computing the difference between a grid azimuth and a geographic azimuth.

The spherical coordinates of the station may be scaled from the map or may be computed. Where the spherical coordinates are known the tables are used in the same manner as that described under the heading "Transformation of geographic to grid azimuths," but when the azimuths are reckoned clockwise the difference will be added when to the east of the central meridian and subtracted when to the west. Note especially the caution in the last paragraph under the preceding heading.

TRANSFORMATION OF GEOGRAPHIC TO GRID AZIMUTHS AND THE REVERSE, APPROXIMATE METHOD.

When only an approximate difference between the grid and spherical azimuths is needed, it may be computed from the x and y coordinates given in the grid tables.

The process would be to take the difference between the x coordinate of the nearest intersection of one minute of longitude and latitude and the x coordinate of an intersection of a higher latitude with the same minute of longitude and divide this difference by the difference in the y coordinates of the two intersections. The quotient would be the tangent of the angle between a grid meridian and a geographic meridian. For instance, the difference in the y coordinates of the intersections of 70° 00′ with 42° 55′ and 42° 59′ is 8100 yards. The difference in the x coordinates is 289 yards. The x difference divided by the y difference would be 0.03568. This is the natural tangent of 2° 02′ 40″.

This angle is added to or subtracted from the spherical azimuth to obtain the grid azimuth, or is applied to the grid azimuth to obtain the spherical azimuth. In this particular case it must be subtracted from the spherical azimuth to get the grid azimuth.

The angle will be correct within one-half minute in all cases.

TRANSFORMATION OF GEOGRAPHIC TO GRID COORDINATES.

Let P represent latitude,

L represent longitude,

C be the station,

A be point on first full minute to the west of C of same latitude as C,

B be point on first full minute to the east of C of same latitude as C,

x_1 and y_1 be grid coordinates of minute intersection southwest of C,

x_2 and y_2 be grid coordinates of minute intersection northwest of C,

x_3 and y_3 be grid coordinates of minute intersection northeast of C,

x_4 and y_4 be grid coordinates of minute intersection southeast of C,

C_P = seconds of latitude of C divided by 60,

C_L = seconds of longitude of C divided by 60.

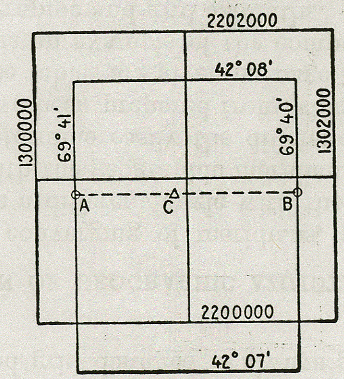

FIG. 5.—TRANSFORMATION OF GEOGRAPHIC TO GRID COORDINATES.

Then we have the following equations:

Grid x coordinate of $A = x_a = x_1 + C_P(x_2 - x_1)$.

x coordinate of $B = x_b = x_4 + C_P(x_3 - x_4)$.

x coordinate of $C = x_c = x_b + C_L(x_a - x_b)$.

y coordinate of $A = y_a = y_1 + C_P(y_2 - y_1)$.

y coordinate of $B = y_b = y_4 + C_P(y_3 - y_4)$.

y coordinate of $C = y_c = y_b + C_L(y_a - y_b)$.

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COMPUTATION OF ARTILLERY GRID COORDINATES FROM GEOGRAPHIC COORDINATES.

[Station Browne. Coordinates are given in yards. The numbers inclosed in parentheses in column 2 represent the values in column 3 opposite the corresponding numbers in column 1.]

1	P	40° 50′ 30″.73	15	x_a-x_b	−1537.2
2	C_P	.512	16	$x=x_b+(4)\times(15)$	927 429.2
3	L	73° 47′ 12″.57	17	y_1	2 040 817.5
4	C_L	.210	18	y_2	2 042 841.6
5	x_1	926 205.3	19	y_3	2 042 827.7
6	x_2	926 223.8	20	y_4	2 040 803.6
7	x_3	927 760.8	21	y_2-y_1	2024.1
8	x_4	927 742.7	22	y_3-y_4	2024.1
9	x_2-x_1	18.5	23	$C_P(y_2-y_1)$	1036.3
10	x_3-x_4	18.1	24	$C_P(y_3-y_4)$	1036.3
11	$C_P(x_2-x_1)$	9.5	25	$y_a=y_1+(23)$	2 041 853.8
12	$C_P(x_3-x_4)$	9.3	26	$y_b=y_4+(24)$	2 041 839.9
13	$x_a=x_1+(11)$	926 214.8	27	y_a-y_b	13.9
14	$x_b=x_4+(12)$	927 752.0	28	$y=y_b+(4)\times(27)$	2 041 842.8

TRANSFORMATION OF GRID TO GEOGRAPHIC COORDINATES.

In figure 6 let 1, 2, 3, and 4 be a quadrangle of minute intersections within which the given point P lies.

Let $x\ y$ be the grid coordinates of P.

Let x_1, x_2, x_3, and x_4 be the x coordinates of the corners 1, 2, 3, and 4, respectively. Let y_1, y_2, y_3, and y_4 be the y coordinates of the corners 1, 2, 3, and 4, respectively.

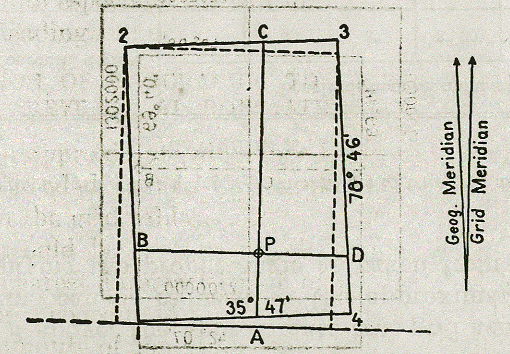

FIG. 6.—TRANSFORMATION OF GRID TO GEOGRAPHIC COORDINATES.

A, B, C, and D are points at which grid lines through P intersect the lines of the minute quadrangle.

Let x_a, x_b, x_c, x_d, y_a, y_b, y_c, and y_d be the grid coordinates of the points A, B, C, and D.

Then

$$y_a=y_4+\frac{x_4-x}{x_4-x_1}(y_1-y_4)$$

$$y_c=y_3+\frac{x_3-x}{x_3-x_2}(y_2-y_3).$$

Then the seconds of latitude of P above the minute line 1–4 will be:

$$\text{Seconds in latitude}=60''\left(\frac{y-y_a}{y_c-y_a}\right).$$

$$x_b=x_1+\frac{y-y_1}{y_2-y_1}(x_2-x_1)$$

$$x_d=x_4+\frac{y-y_4}{y_3-y_4}(x_3-x_4).$$

Then the seconds of longitude of P west of the minute line 3–4 will be:

$$\text{Seconds in longitude}=60''\left(\frac{x_d-x}{x_d-x_b}\right).$$

The computation can best be made on a form similar to the following sample. The grid coordinates of the point P are known, while the grid coordinates of the four-minute intersections are obtained from the grid tables. Care should be taken regarding the signs of differences shown in the form.

COMPUTATION OF GEOGRAPHIC COORDINATES FROM GRID COORDINATES.

[Station Cary. Coordinates are given in yards. The numbers inclosed in parentheses in column 2 represent the values in column 3 opposite the corresponding numbers of column 1.]

1	Lat. minus to south	35° 47′	21	$y-y_A$	459.4
2	Long. minus to east	78° 46′	22	y_C-y_A	2023.9
3	x	1 220 027.5	23.	Sec. lat.$=60''\frac{(21)}{(22)}$	13.62″
4	y	1 430 414.5	24	$y-y_1$	479.2
5	x_1	1 219 154.9	25	y_2-y_1	2022.8
6	x_2	1 219 109.2	26	x_2-x_1	−45.7
7	x_3	1 220 756.3	27	$x_B=(5)+\frac{(24)}{(25)}(26)$	1 219 144.1
8	x_4	1 220 802.4	28	$y-y_4$	441.8
9	y_1	1 429 935.3	29	y_3-y_4	2022.8
10	y_2	1 431 958.1	30	x_3-x_4	−46.1
11	y_3	1 431 995.5	31	$x_D=(8)+\frac{(28)}{(29)}(30)$	1 220 792.3
12	y_4	1 429 972.7	32	x_D-x	764.8
13	x_4-x	774.9	33	x_D-x_B	1648.2
14	x_4-x_1	1647.5	34	Sec. long.$=60''\frac{(32)}{(33)}$	27.84″
15	y_1-y_4	−37.4	35	Lat. $=(1)+(23)$	35° 47′ 13.62″
16	$y_A=(12)+\frac{(13)}{(14)}(15)$	1 429 955.1	36	Long.$=(2)+(34)$	78° 46′ 27.84″
17	x_3-x	728.8			
18	x_3-x_2	1647.1			
19	y_2-y_3	−37.4			
20	$y_C=(11)+\frac{(17)}{(18)}(19)$	1 431 979.0			

REDUCTION OF MAP GRID AZIMUTHS TO TRUE GEOGRAPHIC AZIMUTHS.

The map grid azimuth, denoted by β, is determined by the formula

$$\tan\beta=\frac{x_2-x_1}{y_2-y_1}$$

By applying the proper correction for the reduction from grid azimuth to geographic azimuth (see table on page 33) the approximate geographic azimuth can be found. By interpolation in the table below with this azimuth as one argument and the latitude and longitude of the point observed as the other arguments the proper correction to this approximate azimuth can be found. This is the angular distortion of the projection. When this correction is added to the approximate angle from true north or south, the angle being always taken less than 90°, the true geographic azimuth from this point is obtained.

If the difference of the y's is corrected for the error in scale before computing tan β above, the approximate true grid azimuth will be obtained, and this can be reduced to the approximate geographic azimuth by applying the proper angle as taken from the table on page 33.

Corrections for the reduction of map grid azimuths to true grid azimuths.

LATITUDE 25°.

Angle from true north or south, degrees.	Longitude from central meridian.					
	1°	2°	3°	3° 30′	4°	4° 30′
	″	″	″	″	″	″
0	0.0	0.0	0.0	0.0	0.0	0.0
10	4.4	17.6	39.7	54.0	70.5	89.1
20	8.3	33.1	74.6	101.5	132.5	167.5
30	11.2	44.6	100.5	136.8	178.5	225.8
40	12.7	50.8	114.3	155.6	203.1	256.9
45	12.9	51.6	116.1	158.0	206.3	260.9
50	12.7	50.8	114.3	155.6	203.2	257.0
60	11.2	44.6	100.5	136.9	178.7	226.1
70	8.3	33.1	74.6	101.6	132.7	167.9
80	4.4	17.6	39.7	54.1	70.6	89.3
90	0.0	0.0	0.0	0.0	0.0	0.0

LATITUDE 30°.

0	0.0	0.0	0.0	0.0	0.0	0.0
10	4.0	16.1	36.2	49.3	64.4	81.4
20	7.6	30.3	68.1	92.6	121.0	153.0
30	10.2	40.8	91.8	124.8	163.0	206.2
40	11.6	46.4	104.4	142.0	185.5	234.6
45	11.8	47.1	106.0	144.2	188.4	238.3
50	11.6	46.4	104.4	142.0	185.5	234.7
60	10.2	40.8	91.8	124.9	163.2	206.5
70	7.6	30.3	68.1	92.7	121.2	153.3
80	4.0	16.1	36.2	49.3	64.5	81.6
90	0.0	0.0	0.0	0.0	0.0	0.0

LATITUDE 35°.

0	0.0	0.0	0.0	0.0	0.0	0.0
10	3.6	14.4	32.4	44.1	57.6	72.9
20	6.8	27.1	61.0	82.9	108.2	137.0
30	9.1	36.5	82.1	111.7	145.9	184.6
40	10.4	41.5	93.4	127.1	165.9	210.0
45	10.5	42.2	94.8	129.0	168.5	213.3
50	10.4	41.5	93.4	127.1	166.0	210.1
60	9.1	36.5	82.1	111.8	146.0	184.8
70	6.8	27.1	61.0	83.0	108.4	137.2
80	3.6	14.4	32.5	44.2	57.7	73.0
90	0.0	0.0	0.0	0.0	0.0	0.0

Corrections for the reduction of map grid azimuths to true grid azimuths—Continued.

LATITUDE 40°.

Angle from true north or south, degrees.	Longitude from central meridian.					
	1°	2°	3°	3° 30′	4°	4° 30′
	″	″	″	″	″	″
0	0.0	0.0	0.0	0.0	0.0	0.0
10	3.1	12.6	28.3	38.6	50.4	63.7
20	5.9	23.7	53.3	72.5	94.7	119.8
30	7.9	32.0	71.8	97.7	127.6	161.4
40	9.0	36.3	81.6	111.1	145.1	183.6
45	9.2	36.9	82.9	112.9	147.4	186.5
50	9.0	36.4	81.6	111.2	145.1	183.7
60	8.0	32.0	71.8	97.8	127.7	161.6
70	5.9	23.7	53.3	72.6	94.8	120.0
80	3.1	12.6	28.4	38.6	50.4	63.8
90	0.0	0.0	0.0	0.0	0.0	0.0

LATITUDE 45°.

0	0.0	0.0	0.0	0.0	0.0	0.0
10	2.7	10.8	24.1	32.9	42.9	54.3
20	5.0	20.2	45.4	61.8	80.7	102.1
30	6.8	27.2	61.2	83.3	108.7	137.6
40	7.7	31.0	69.5	94.7	123.6	156.5
45	7.8	31.5	70.6	96.2	125.5	158.9
50	7.7	31.0	69.6	94.7	123.6	156.5
60	6.8	27.2	61.2	83.3	108.8	137.7
70	5.0	20.2	45.4	61.8	80.7	102.2
80	2.7	10.8	24.2	32.9	43.0	54.4
90	0.0	0.0	0.0	0.0	0.0	0.0

LATITUDE 50°.

0	0.0	0.0	0.0	0.0	0.0	0.0
10	2.2	8.9	20.0	27.2	35.5	44.9
20	4.2	16.7	37.5	51.1	66.7	84.4
30	5.6	22.5	50.5	68.8	89.9	113.7
40	6.4	25.6	57.5	78.3	102.2	129.3
45	6.5	26.0	58.4	79.5	103.8	131.3
50	6.4	25.6	57.5	78.3	102.2	129.3
60	5.6	22.5	50.6	68.9	89.9	113.8
70	4.2	16.7	37.5	51.1	66.7	84.4
80	2.2	8.9	20.0	27.2	35.5	44.9
90	0.0	0.0	0.0	0.0	0.0	0.0

FORMULAS FOR THE COMPUTATION OF COORDINATES ON THE ORDINARY OR AMERICAN POLYCONIC PROJECTION.

The latitude is denoted by ϕ and the longitude out from the central meridian by λ.

$$x' = \frac{a \cot \phi}{(1-\epsilon^2 \sin^2 \phi)^{1/2}} \sin (\lambda \sin \phi).$$

$$y' = \frac{a \cot \phi}{(1-\epsilon^2 \sin^2 \phi)^{1/2}} [1 - \cos (\lambda \sin \phi)] = \frac{2a \cot \phi}{(1-\epsilon^2 \sin^2 \phi)^{1/2}} \sin^2 \left(\frac{\lambda \sin \phi}{2}\right).$$

$$\text{Tabular } x = 1\,000\,000.0 + x'$$

$$\text{Tabular } y = 2\,000\,000.0 + y_1 + y'$$

y_1 is the meridional arc from the parallel of 40° 30′ to the given parallel. 2 000 000.0 + y_1 is equal to the value of y on the central meridian.

$$\rho_n = \frac{a}{(1-\epsilon^2 \sin^2 \phi)^{1/2}}$$

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ρ_n in meters is connected with the A factor in Special Publication No. 8, United States Coast and Geodetic Survey, by the relation

$$\log \rho_n = \text{colog } A + \text{colog } \sin 1''$$
$$\log \rho_n \text{ in yards} = \text{colog } A + \text{colog } \sin 1'' + 0.0388629.$$

When λ does not exceed 4° 30′, the following approximations can be used:

$$x' = \rho_n \lambda \cos \phi \left(1 - \frac{1}{6}\lambda^2 \sin^2\phi\right)$$

$$y' = \frac{1}{4}\rho_n \lambda^2 \sin 2\phi \left(1 - \frac{1}{12}\lambda^2 \sin^2\phi\right)$$

Because of the fact that interpolations were employed in the computation of the table, the values resulting from rigid computation will differ slightly from the tabular values. These differences should in no case to be greater than 1 yard.

$$\text{Scale error along the meridian} = \frac{1}{2}\lambda^2 \cos^2 \phi.$$

If η is the angle formed by the intersection of the meridian with the parallel on the side facing the central meridian, we have

$$\psi = \eta - \frac{\pi}{2} = \frac{1}{12}\lambda^3 \sin 2\phi \cos \phi.$$

In the last four formulas λ must be given in units of circular measure or radians.

TRANSFORMATION OF LOCAL PLANE COORDINATES TO STANDARD GRID COORDINATES.

In the triangulation of Greater New York[1] the positions of the stations are given by local plane coordinates, that is, by their distances north or south and east or west from some arbitrary local origin. A test has been made to see if a direct transformation can be made from these local plane coordinates to the grid coordinates without having to compute or make use of the geographic positions of the stations. Prescott Water Tower, one of the stations selected for the test, is the origin of the plane coordinates for a part of the New York triangulation. Payne is the name of the other station used in the test.

From an interpolation of the tables for the "Reduction of geodetic azimuths to grid azimuths" (see p. 33) the value of α (which is the angle between the meridian and the grid line, at Prescott Water Tower) is found to be 37′ 50.3″. The geodetic position of Prescott Water Tower is as follows: Lat. 40° 40′ 20.721″, long. 73° 58′ 03.841″.

[1] See Report on the Triangulation of Greater New York by the cooperation of the City of New York and the U. S. Coast and Geodetic Survey. Published by the City of New York.

This value of α would be used for every transformation of coordinates in which Prescott Water Tower is taken as the origin of coordinates ($x=0$, $y=0$). However, for each origin a new α would be figured.

If the local origin lies east of the central meridian, the angle α should be considered negative, which would merely change the sign of the terms containing $\sin \alpha$.

Computations were then made to determine whether there would be an appreciable difference in the grid values of station Payne when figured by two methods: First method—Plane coordinates of Payne, holding Prescott Water Tower as an origin with the Y axis parallel to the seventy-third meridian were computed, and the values so determined were added to the computed grid coordinates of Prescott Water Tower; the summation of the two gave the grid coordinates of Payne with respect to the standard grid system. Second method—Grid coordinates of Payne were figured directly from the geodetic position of Payne, with respect to the standard system in the usual manner.

First method.—The computation of the grid coordinates of Payne with Prescott Water Tower as origin is as follows: For the transformation of the coordinates, with respect to the grid line through the local origin, the following formulas are used:

$$x' = x\cos\alpha + y\sin\alpha$$
$$y' = -x\sin\alpha + y\cos\alpha$$

x and y are the old local coordinates.

x' and y' are local coordinates referred to local origin but with the Y axis parallel to the seventy-third meridian.

For station Payne:

x	=22 916. 16 yards.	y	=10 387. 999 yards.
Log x	= 4. 3601418	Log y	= 4. 0165319
Log cos α =	9. 9999737	Log sin α =	8. 0416780
	14. 3601155 − 10		12. 0582099 − 10
$x \cos \alpha$	=22 914. 76 yds.	$y \sin \alpha$	= 114. 343 yds.

$x' = 22\,914.76 + 114.343 = 23\,029.10$ yards for Payne (origin Prescott Water Tower).

Log x	= 4. 3601418	Log y	= 4. 0165319
Log sin α =	8. 0416780	Log cos α =	9. 9999737
	12. 4018198 − 10		14. 0165056 − 10
$-x \sin \alpha$	= − 252. 243	$y \cos \alpha$	=10 387. 4

$y' = 10\,387.4 - 252.24 = 10\,135.16$ for Payne (origin Prescott Water Tower).

The computation of the standard grid coordinates from geographic coordinates for Prescott Water Tower (the local origin) was then made

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in the usual manner, and the above values determined for Payne station were added to the standard grid coordinates of Prescott Water Tower to determine the standard grid values of Payne.

Standard grid values for Prescott Water Tower $\begin{cases} x = 910\,518.0 \\ y = 2\,021\,431.2 \end{cases}$

$x_{PWT} =$ 910 518. 0
$x'_{Payne} =$ 23 029. 1 (referred to Prescott Water Tower).

$x_{Payne} =$ 933 547. 1 (standard grid value).
$y_{PWT} =$ 2 021 431. 2
$y'_{Payne} =$ 10 135. 2 (referred to Prescott Water Tower).

$y_{Payne} =$ 2 031 566. 4 standard grid value.

Second method.—Computations were then made for Payne in the usual manner to determine directly from its geodetic position the values of the standard grid coordinates.

Values determined are as follows:

Payne $\begin{cases} \text{Lat. } 40° \; 45' \; 27.701''. \\ \text{Long. } 73° \; 43' \; 10.517''. \end{cases}$

$x =$ 933 547.2 yards
$y =$ 2 031 567.1 yards } Correct values on standard grid.

The y value as determined by the first method may now be corrected for scale error. For station Payne (Lat. 40° 45′ 27.701″—Long. 73° 43′ 10.517″) the scale error in yards per thousand yards, to be applied to the y' value as determined in first method, is found from the table of "corrections to y coordinates for magnification of scale" to be approximately 0.05 yard per thousand yards.

y'_{Payne}	= 10.1352 thousand yards	
Correction to be applied	= 10.1352 × 0.05 yd.	= 0.5 yard
y'_{Payne} (corrected)	= 10 135.2 + 0.5	= 10 135.7
y Prescott Water Tower	(standard grid)	= 2 021 431.2
y_{Payne} (standard grid)		= 2 031 566.9

Difference between the y value (standard grid) of Payne as determined by the two methods, the scale correction having been applied to the value obtained in the first method, equals −0.2 yard.

y value (standard grid) computed directly from the geodetic position of Payne = 2 031 567.1 yards.

y value (standard grid) computed about Prescott Water Tower as origin, with correction for scale error = 2 031 566.9 yards.

Difference = −0.2 yard.

The x values as determined by the two methods differ by +0.1 yard.

By applying the correction to the y value the grid coordinates, as computed about Prescott Water Tower as an origin and then transferred to the standard grid, lie well within the range of precision required.

COMPUTATION FOR THE TRANSFORMATION OF LOCAL PLANE COORDINATES TO STANDARD GRID COORDINATES.

[Station Payne. Coordinates are given in yards. The numbers inclosed in parentheses in column 2 represent the values in column 3 opposite the corresponding numbers in column 1.]

1	x_1	910 518. 0	14	$x' = (11) + (13)$	23 029. 10
2	y_1	2 021 431. 2	15	Log x sin α	2. 4018198
3	α	00° 37′ 50.3″	16	$-x$ sin α	−252. 243
4	x	22 916. 16	17	Log y cos α	4. 0165056
5	y	10 388. 00	18	y cos α	10 387. 4
6	Log x	4. 3601418	19	$y' = (16) + (18)$	+10 135. 16
7	Log y	4. 0165319	20	Scale error per 1000 yards	0. 05
8	Log sin α	8. 0416780			
9	Log cos α	9. 9999737	21	Scale corr. to y'	+0. 5
10	Log x cos α	4. 3601155	22	$y_1 = (19) + (21)$	+10 135. 7
11	x cos α	22 914. 76	23	Grid $x = (1) + (14)$	933 547. 1
12	Log y sin α	2. 0582099	24	Grid $y = (2) + (22)$	2 031 566. 9
13	y sin α	114. 343			

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Example of grid table for one-minute intersections.

Zone.	Long.	Lat. 42° 55′. x	Lat. 42° 55′. y	Lat. 42° 56′. x	Lat. 42° 56′. y	Lat. 42° 57′. x	Lat. 42° 57′. y	Lat. 42° 58′. x	Lat. 42° 58′. y	Lat. 42° 59′. x	Lat. 42° 59′. y
	° ′	Yards.	Yards.	Yards.	Yards.	Yards.	Yards.	Yards.	Yards.	Yards.	Yards.
A B C D E F G	70 78 86 94 102 110 118 } 00	1 267 834.3	2 298 311.1	1 267 762.1	2 300 336.1	1 267 689.8	2 302 361.1	1 267 617.5	2 304 386.1	1 267 545.3	2 306 411.2
	01	66 346.9	258.2	66 275.1	283.2	66 203.2	308.2	66 131.3	333.2	66 059.5	358.3
	02	64 859.6	205.6	64 788.1	230.6	64 716.6	255.6	64 645.2	280.6	64 573.7	305.7
	03	63 372.2	153.3	63 301.2	178.3	63 230.1	203.3	63 159.0	228.3	63 088.0	253.4
	04	61 884.9	101.3	61 814.2	126.3	61 743.5	151.3	61 672.9	176.3	61 602.2	201.4
	05	60 397.5	2 298 049.6	60 327.2	074.6	60 256.9	099.5	60 186.7	124.5	60 116.4	149.6
	06	58 910.1	2 297 998.2	58 840.2	2 300 023.2	58 170.3	2 302 048.1	58 700.5	073.1	58 630.6	098.2
	07	57 422.7	947.1	57 353.2	2 299 972.1	57 283.7	2 301 997.0	57 214.3	2 304 022.0	57 144.8	2 306 047.1
	08	55 935.2	896.2	55 866.2	921.2	55 797.1	946.1	55 728.0	2 303 971.1	55 659.0	2 305 996.2
	09	54 447.8	845.7	54 379.2	870.7	54 310.5	895.6	54 241.8	920.6	54 173.2	945.7
	10	1 252 960.4	2 297 795.4	1 252 892.2	2 299 820.4	1 252 823.9	2 301 845.3	1 252 755.6	2 303 870.3	1 252 687.4	2 305 895.4
	11	51 472.9	745.5	51 405.1	770.4	51 337.2	795.4	51 269.3	820.4	51 201.5	845.4
	12	49 985.4	695.8	49 918.0	720.8	49 850.6	745.7	49 783.1	770.7	49 715.6	795.8
	13	48 498.0	646.5	48 431.0	671.4	48 363.9	696.4	48 296.8	721.4	48 229.8	746.4
	14	47 010.5	597.4	46 943.9	622.3	46 877.3	647.3	46 810.6	672.3	46 743.9	697.3
	15	45 523.0	548.6	45 456.8	573.5	45 390.6	598.5	45 324.3	623.5	45 258.0	648.5
	16	44 035.5	500.1	43 969.7	525.0	43 904.0	550.0	43 838.0	575.0	43 772.1	600.0
	17	42 548.0	451.9	42 482.6	476.8	42 417.2	501.8	42 351.7	526.8	42 286.2	551.8
	18	41 060.5	404.1	40 995.4	429.0	40 930.4	454.0	40 865.4	479.0	40 800.3	504.0
	19	39 573.0	356.5	39 508.3	381.4	39 443.7	406.4	39 379.1	431.4	39 314.4	456.4
	20	1 238 085.5	2 297 309.1	1 238 021.2	2 299 334.1	1 237 957.0	2 301 359.1	1 237 892.8	2 303 384.0	1 237 828.5	2 305 409.0
	21	36 597.9	262.1	36 534.0	287.1	36 470.2	312.1	36 406.4	337.0	36 342.5	362.0
	22	35 110.4	215.5	35 046.9	240.4	34 983.5	265.3	34 920.1	290.3	34 856.6	315.4
	23	33 622.8	169.0	33 559.7	193.9	33 496.7	218.9	33 433.7	243.9	33 370.6	268.9
	24	32 135.3	123.7	32 072.6	148.7	32 010.0	173.7	31 947.4	198.6	31 884.7	223.6
	25	30 647.7	077.1	30 585.4	102.0	30 523.2	126.9	30 461.0	151.9	30 398.7	177.0
	26	29 160.1	2 297 031.5	29 098.2	056.5	29 036.4	081.5	28 974.6	106.4	28 912.7	131.4
	27	27 672.5	2 296 986.3	27 611.0	2 299 011.2	27 549.6	2 301 036.1	27 488.2	061.1	27 426.7	086.2
	28	26 184.9	941.3	26 123.8	2 298 966.3	26 062.8	2 300 991.3	26 001.8	2 303 016.2	25 940.7	2 305 041.2
	29	1 224 697.3	2 296 896.7	1 224 636.6	2 298 921.7	1 224 576.0	2 300 946.6	1 224 515.4	2 302 971.5	1 224 454.7	2 304 996.6

Current Miscellaneous File 106

Corrections to y coordinates for magnification of scale.

[Correction is in yards per thousand yards.]

Latitude.	Zone and longitude.									
	° ′ A, 68 30 B, 76 30 C, 84 30 D, 92 30 E, 100 30 F, 108 30 G, 116 30	° 69 77 85 93 101 109 117	° ′ 69 30 77 30 85 30 93 30 101 30 109 30 117 30	° 70 78 86 94 102 110 118	° ′ 70 30 78 30 86 30 94 30 102 30 110 30 118 30	° 71 79 87 95 103 111 119	° ′ 71 30 79 30 87 30 95 30 103 30 111 30 119 30	° 72 80 88 96 104 112 120	° ′ 72 30 80 30 88 30 96 30 104 30 112 30 120 30	° 73 81 89 97 105 113 121
° 24	2.574	2.034	1.557	1.144	0.794	0.508	0.286	0.127	0.032	0.000
25	2.533	2.002	1.533	1.126	.782	.500	.281	.125	.031	.000
26	2.492	1.969	1.507	1.107	.769	.492	.277	.123	.031	.000
27	2.449	1.935	1.481	1.088	.756	.484	.272	.121	.030	.000
28	2.404	1.900	1.455	1.069	.742	.475	.267	.119	.030	.000
29	2.359	1.864	1.427	1.049	.728	.466	.262	.117	.029	.000
30	2.313	1.828	1.399	1.028	.714	.457	.257	.114	.029	.000
31	2.266	1.790	1.371	1.007	.699	.448	.252	.112	.028	.000
32	2.218	1.753	1.342	0.986	.685	.438	.246	.110	.027	.000
33	2.169	1.714	1.312	.964	.670	.429	.241	.107	.027	.000
34	2.120	1.675	1.282	.942	.654	.419	.236	.105	.026	.000
35	2.070	1.635	1.252	.920	.639	.409	.230	.102	.026	.000
36	2.018	1.595	1.221	.897	.623	.399	.224	.100	.025	.000
37	1.967	1.554	1.190	.874	.607	.389	.219	.097	.024	.000
38	1.915	1.513	1.159	.851	.591	.378	.213	.095	.024	.000
39	1.863	1.472	1.127	.828	.575	.368	.207	.092	.023	.000
40	1.810	1.430	1.095	.804	.559	.358	.201	.089	.022	.000
41	1.757	1.388	1.063	.781	.542	.347	.195	.087	.022	.000
42	1.703	1.346	1.030	.757	.526	.336	.189	.084	.021	.000
43	1.650	1.303	0.997	.733	.509	.326	.183	.081	.020	.000
44	1.596	1.261	.965	.709	.493	.315	.177	.079	.020	.000
45	1.542	1.218	.933	.685	.476	.305	.171	.076	.019	.000
46	1.488	1.176	.900	.661	.459	.294	.165	.073	.018	.000
47	1.435	1.133	.868	.638	.443	.283	.159	.071	.018	.000
48	1.381	1.091	.835	.614	.426	.273	.153	.068	.017	.000
49	1.328	1.049	.803	.590	.410	.262	.148	.066	.016	.000
50	1.274	1.007	.771	.566	.393	.252	.142	.063	.016	.000

Latitude.	Zone and longitude—Continued.									
	° A, 73 B, 81 C, 89 D, 97 E, 105 F, 113 G, 121	° ′ 73 30 81 30 89 30 97 30 105 30 113 30 121 30	° 74 82 90 98 106 114 122	° ′ 74 30 82 30 90 30 98 30 106 30 114 30 122 30	° 75 83 91 99 107 115 123	° ′ 75 30 83 30 91 30 99 30 107 30 115 30 123 30	° 76 84 92 100 108 116 124	° ′ 76 30 84 30 92 30 100 30 108 30 116 30 124 30	° 77 85 93 101 109 117 125	° ′ 77 30 85 30 93 30 101 30 109 30 117 30 125 30
° 24	0.000	0.032	0.127	0.286	0.508	0.794	1.144	1.557	2.034	2.574
25	.000	.031	.125	.281	.500	.782	1.126	1.533	2.002	2.533
26	.000	.031	.123	.277	.492	.769	1.107	1.507	1.969	2.492
27	.000	.030	.121	.272	.484	.756	1.088	1.481	1.935	2.449
28	.000	.030	.119	.267	.475	.742	1.069	1.455	1.900	2.404
29	.000	.029	.117	.262	.466	.728	1.049	1.427	1.864	2.359
30	.000	.029	.114	.257	.457	.714	1.028	1.399	1.828	2.313
31	.000	.028	.112	.252	.448	.699	1.007	1.371	1.790	2.266
32	.000	.027	.110	.246	.438	.685	0.986	1.342	1.753	2.218
33	.000	.027	.107	.241	.429	.670	.964	1.312	1.714	2.169
34	.000	.026	.105	.236	.419	.654	.942	1.282	1.675	2.120
35	.000	.026	.102	.230	.409	.639	.920	1.252	1.635	2.070
36	.000	.025	.100	.224	.399	.623	.897	1.221	1.595	2.018
37	.000	.024	.097	.219	.389	.607	.874	1.190	1.554	1.967
38	.000	.024	.095	.213	.378	.591	.851	1.159	1.513	1.915
39	.000	.023	.092	.207	.368	.575	.828	1.127	1.472	1.863
40	.000	.022	.089	.201	.358	.559	.804	1.095	1.430	1.810
41	.000	.022	.087	.195	.347	.542	.781	1.063	1.388	1.757
42	.000	.021	.084	.189	.336	.526	.757	1.030	1.346	1.703
43	.000	.020	.081	.183	.326	.509	.733	0.997	1.303	1.650
44	.000	.020	.079	.177	.315	.493	.709	.965	1.261	1.596
45	.000	.019	.076	.171	.305	.476	.685	.933	1.218	1.542
46	.000	.018	.073	.165	.294	.459	.661	.900	1.176	1.488
47	.000	.018	.071	.159	.283	.443	.638	.868	1.133	1.435
48	.000	.017	.068	.153	.273	.426	.614	.835	1.091	1.381
49	.000	.016	.066	.148	.262	.410	.590	.803	1.049	1.328
50	.000	.016	.063	.142	.252	.393	.566	.771	1.007	1.274

74704

Corrections to y coordinates for magnification of scale—Continued.

EXTENSION OF TABLE FOR EASTERN MAINE.

Latitude.	Longitude.			
	66° 30′	67° 00′	67° 30′	68° 00′
°				
44	3.330	2.837	2.384	1.970
45	3.218	2.742	2.304	1.904
46	3.105	2.646	2.223	1.837
47	....	2.550	2.143	1.771
48	....	2.455	2.063	1.705

Approximate corrections to y coordinates for magnification of scale.

[Correction is in yards per thousand yards.]

Latitude.	Longitude from central meridian.													
	0° 00′	0° 30′	1° 00′	1° 30′	2° 00′	2° 30′	3° 00′	3° 30′	4° 00′	4° 30′	5° 00′	5° 30′	6° 00′	6° 30′
°														
24	0.0	0.0	0.1	0.3	0.5	0.8	1.1	1.6	2.0	2.6	......	......	......	......
26	.0	.0	.1	.3	.5	.8	1.1	1.5	2.0	2.5	......	......	......	......
28	.0	.0	.1	.3	.5	.7	1.1	1.5	1.9	2.4	......	......	......	......
30	.0	.0	.1	.3	.5	.7	1.0	1.4	1.8	2.3	......	......	......	......
32	.0	.0	.1	.2	.4	.7	1.0	1.3	1.8	2.2	......	......	......	......
34	.0	.0	.1	.2	.4	.7	0.9	1.3	1.7	2.1	......	......	......	......
36	.0	.0	.1	.2	.4	.6	.9	1.2	1.6	2.0	......	......	......	......
38	.0	.0	.1	.2	.4	.6	.9	1.2	1.5	1.9	......	......	......	......
40	.0	.0	.1	.2	.4	.6	.8	1.1	1.4	1.8	......	......	......	......
	.0	.0	.1	.2	.3	.5	.8	1.0	1.3	1.7	......	......	......	......
44	.0	.0	.1	.2	.3	.5	.7	1.0	1.3	1.6	2.0	2.4	2.8	3.3
46	.0	.0	.1	.2	.3	.5	.7	0.9	1.2	1.5	1.8	2.2	2.6	3.1
48	.0	.0	.1	.2	.3	.4	.6	.8	1.1	1.4	1.7	2.1	2.5	......
50	.0	.0	.1	.1	.3	.4	.6	.8	1.0	1.3	......	......	......	......

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Corrections for the reduction of geographic azimuths to grid azimuths.

[Tabular value $= \lambda \sin \phi - \frac{1}{12} \lambda^3 \left(\frac{\pi}{180}\right)^2 \sin 2\phi \cos \phi$; λ = distance in degrees from the central meridian; ϕ = latitude.]

Latitude.	Zone and longitude.											
	° 67	° A, 68 B, 76 C, 84 D, 92 E, 100 F, 108 G, 116	° 69 77 85 93 101 109 117	° 70 78 86 94 102 110 118	° 71 79 87 95 103 111 119	° 72 80 88 96 104 112 120	° 73 81 89 97 105 113 121	° 74 82 90 98 106 114 122	° 75 83 91 99 107 115 123	° 76 84 92 100 108 116 124	° 77 85 93 101 109 117 125	° 78 86 94 102 110 118 126
°	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″
24	....	2 01 53	1 37 33	1 13 11	0 48 48	0 24 24	0 00 00	0 24 24	0 48 48	1 13 11	1 37 33	2 01 53
25	....	06 39	41 22	16 03	50 42	25 21	00	25 21	50 42	16 03	41 22	06 39
26	....	11 23	45 08	18 53	52 36	26 18	00	26 18	52 36	18 53	45 08	11 23
27	....	16 04	48 53	21 41	54 28	27 14	00	27 14	54 28	21 41	48 53	16 04
28	....	20 42	52 36	24 28	56 20	28 10	00	28 10	56 20	24 28	52 36	20 42
29	....	25 18	56 17	27 14	58 10	29 05	00	29 05	58 10	27 14	56 17	25 18
30	....	29 51	1 59 56	29 58	0 59 59	30 00	00	30 00	0 59 59	29 58	1 59 56	29 51
31	....	34 22	2 03 32	32 40	1 01 48	30 54	00	30 54	1 01 48	32 40	2 03 32	34 22
32	....	38 50	07 06	35 21	03 35	31 48	00	31 48	03 35	35 21	07 06	38 50
33	....	43 15	10 38	38 00	05 21	32 41	00	32 41	05 21	38 00	10 38	43 15
34	....	47 37	14 08	40 37	07 06	33 33	00	33 33	07 06	40 37	14 08	47 37
35	....	51 56	17 35	43 13	08 49	34 25	00	34 25	08 49	43 13	17 35	51 56
36	....	2 56 11	21 00	45 46	10 31	35 16	00	35 16	10 31	45 46	21 00	2 56 11
37	....	3 00 24	24 22	48 18	12 12	36 06	00	36 06	12 12	48 18	24 22	3 00 24
38	....	04 33	27 41	50 47	13 52	36 56	00	36 56	13 52	50 47	27 41	04 33
39	....	08 39	30 58	53 15	15 30	37 45	00	37 45	15 30	53 15	30 58	08 39
40	....	12 42	34 12	55 40	17 07	38 34	00	38 34	17 07	55 40	34 12	12 42
41	....	16 40	37 23	1 58 04	18 43	39 22	00	39 22	18 43	1 58 04	37 23	16 40
42	....	20 36	40 31	2 00 25	20 17	40 09	00	40 09	20 17	2 00 25	40 31	20 36
43	....	24 28	43 36	02 44	21 50	40 55	00	40 55	21 50	02 44	43 36	24 28
44	4 09 50	28 16	46 39	05 00	23 21	41 41	00	41 41	23 21	05 00	46 39	28 16
45	14 19	32 00	49 38	07 15	24 51	42 25	00	42 25	24 51	07 15	49 38	32 00
46	18 44	35 40	52 34	09 27	26 19	43 10	00	43 10	26 19	09 27	52 34	35 40
47	23 04	39 17	55 27	11 37	27 45	43 53	00	43 53	27 45	11 37	55 27	39 17
48	27 19	42 49	2 58 17	13 44	29 10	44 35	00	44 35	29 10	13 44	2 58 17	42 49
49	....	46 17	3 01 04	15 49	30 33	45 17	00	45 17	30 33	15 49	3 01 04	46 17
50	....	49 42	03 47	17 52	31 55	45 58	00	45 58	31 55	17 52	03 47	49 42

74705

Serial No. 112

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
E. LESTER JONES, SUPERINTENDENT

GRID SYSTEM FOR PROGRESSIVE MAPS IN THE UNITED STATES

By

WILLIAM BOWIE
Chief, Division of Geodesy
Major of Engineers, U. S. Army, 1918-1919

and

OSCAR S. ADAMS
Geodetic Computer

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